Supersymmetry Breaking Vacua from M Theory Fivebranes

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We consider intersecting brane configurations realizing $\mathcal{N} = 2$ supersymmetric gauge theories broken to $\mathcal{N} = 1$ by multitrace superpotentials, and softly to $\mathcal{N} = 0$. We analyze, in the framework of M5-brane wrapping a curve, the supersymmetric vacua and the analogs of spontaneous supersymmetry breaking and soft supersymmetry breaking in gauge theories. We show that the M5-brane does not exhibit the analog of metastable spontaneous supersymmetry breaking, and does not have non-holomorphic minimal volume curves with holomorphic boundary conditions. However, we find that any point in the $\mathcal{N} = 2$ moduli space can be rotated to a non-holomorphic minimal volume curve, whose boundary conditions break supersymmetry. We interpret these as the analogs of soft supersymmetry breaking vacua in the gauge theory.

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1. Introduction and Summary

A promising candidate for new physics beyond the Standard Model is supersymmetry, which offers a solution to the hierarchy problem, a unification of gauge couplings and a dark matter candidate. Supersymmetry is broken in nature and one of the most important problems is to understand the mechanism that leads to this breaking.

One way of breaking supersymmetry is by adding explicit soft supersymmetry breaking terms to the supersymmetric Lagrangian, as in the case of the MSSM. Others are spontaneous supersymmetry breaking mechanisms, which are particularly interesting when supersymmetry is broken dynamically. A new paradigm for dynamical supersymmetry breaking has been advocated by ISS [1], in which the theory contains both supersymmetric vacua and also vacua that break supersymmetry dynamically. In these scenarios, the supersymmetry breaking vacuum is meta-stable. It has recently become clear that this phenomenon is rather generic in supersymmetric gauge theories (for a review and a discussion of recent developments see [2]).

An important question is whether and how these supersymmetry breaking mechanisms can be realized in M/string theory. One framework to address this question is in the intersecting branes setup (for a review see [3]). Here, one typically engineers the gauge theory on the worldvolume of D-branes in the type IIA superstring theory, where additional D-branes and NS5-branes are used in order to get the required amount of supersymmetry, field content and superpotential. This intersecting branes picture provides at low energy and small string coupling limit the classical gauge field theory. One way of analyzing the quantum properties of the system is by lifting a type IIA brane configuration to M theory and realizing it using an M5-brane wrapping a curve. This method has been very successful for analyzing the quantum vacua structure of supersymmetric gauge theories. In these cases the M5-brane is wrapping a holomorphic curve, whose properties encode the supersymmetric vacua structure. This works despite the fact that the M5-brane description is valid for large string coupling, which is the opposite limit to that of the gauge theory one. The reason for this success is the holomorphicity property of the quantities being studied. Indeed, non-holomorphic quantities, such as the Kahler potential and higher derivative couplings, differ between the gauge theory and the M5-brane description [4].

When supersymmetry is broken, the M5-brane is wrapping a non-holomorphic curve of minimal volume (see e.g. [5][6]). However, in this case there is no reason for an agreement between the gauge field theory and the M5-brane description, since their regimes of validity

are very different. Such a disagreement was found, for instance, between the quantum ISS model and the M5-brane description [7][8]. Other examples have been studied in [9][10][11][12][13][14]. [1]

It is clear that if one is interested in the quantum properties of the gauge field theory, the way to proceed is to analyze the intersecting branes configuration in the gauge theory limit. A different study is to analyze supersymmetry breaking in the framework of an M5-brane wrapping a curve. This theory is a six-dimensional one at high-energy and a four-dimensional one at length scales much larger than the typical size of the curve. In this paper, we will analyze supersymmetry breaking in this framework.

As noted above, supersymmetric vacua are realized as an M5-brane wrapping a holomorphic curve. One may define spontaneous supersymmetry breaking vacua as an M5-brane wrapping a non-holomorphic minimal volume curve, which has holomorphic boundary conditions at infinity. Thus, the curve has the same asymptotics as that of a supersymmetric one, but differs in the interior. One may also define an explicit breaking by an M5-brane wrapping a non-holomorphic minimal volume curve, which has non-holomorphic boundary conditions at infinity. Note that these definitions are motivated by the four-dimensional gauge field theory. From the M5-brane theory viewpoint, different minimal volume curves are different choices of vacua, while the high-energy six-dimensional world-volume theory is supersymmetric.

In this paper we will consider intersecting brane configurations realizing $\mathcal{N}=2$ supersymmetric gauge theories broken to $\mathcal{N}=1$ by multitrace superpotentials, and softly to $\mathcal{N}=0$. We will analyze in the M5-brane framework the analogs of spontaneous supersymmetry breaking and soft supersymmetry breaking.

1.1. Summary of the results

We will start by presenting in Section 2, following the work of [21][22], the field theory analysis of pure $\mathcal{N}=2$ SYM with gauge group G, broken to $\mathcal{N}=1$ by the higher trace superpotential for the adjoint

$$W=\sum_{i=1}^{k} s_i \mathrm{Tr}\,\Phi^{i+1}\ , \tag{1.1}$$

[1] In this M5-brane framework one can study also the brane/antibrane configurations in type IIA, or their type IIB dual, which usually do not have a gauge theory limit [15][16][17][18][19][20].

for $k > \operatorname{rank} G$. For a particular choice of couplings s_k, the gauge theory develops a long-lived meta-stable vacuum at the origin of the $\mathcal{N}=2$ Coulomb branch. The existence of this vacuum relies on the exact knowledge of the $\mathcal{N}=2$ Kahler potential.[2]

In Section 3 we will construct the type IIA brane configuration that realizes the classical gauge theory (1.1) by taking k NS5 branes at an angle and suspending D4-branes between them, with more NS5 branes than D4-branes. We will explicitly work out the difference between an $SU(N)$ and $U(N)$ gauge theory. This difference is important since the N D-branes worldvolume gauge group is $U(N)$ and, when $k > \operatorname{rank} G$, the abelian factor, corresponding to the center of mass of the D4-brane stack, plays a fundamental role. The superpotential (1.1) gives rise to k extra supersymmetric brane configurations. We will lift to M theory these supersymmetric vacua, by considering an M5-brane wrapping a holomorphic curve. These are new vacua whose lift is not part of the analysis in [24][25]. Because $k > \operatorname{rank} G$, the M5-brane has several disconnected components, and it successfully reproduces all the gauge theory supersymmetric vacua.

In Section 4, we will consider the meta-stable supersymmetry breaking vacuum in the M5-brane framework. We will look for non-holomorphic minimal volume curve, with holomorphic asymptotic boundary conditions. We will see that there is no such curve, even when we take into account the gravitational backreaction of the disconnected branches of the M5-brane. Thus, the quantum meta-stable gauge theory vacuum is not reproduced in the M5-brane picture.

In Section 5, we will allow the M5-brane curve to have non-holomorphic boundary conditions at infinity of the kind

$$w = m(v + \bar{v}) \ . \tag{1.2}$$

We will find a family of minimal area non-holomorphic genus one curves, whose boundary conditions are parameterized by the modulus τ of the torus, each of which provides a lift of the intersecting branes configurations. We evaluate the action of the M5-brane wrapping this curve, which represents the energy of these vacua. In Section 6 we interpret the boundary condition (1.2) as the analog of soft supersymmetry breaking in the $\mathcal{N}=2$ gauge theory perturbed by

$$\mathcal{L}_{soft} = \int d^4\theta \frac{X^\dagger X}{\Lambda_s^2} u_1^\dagger u_1 + \int d^2\theta M\, u_1^2 + \text{h.c.} \tag{1.3}$$

[2] An example of a metastable vacuum in $\mathcal{N}=2$ gauge theory with flavors and a FI term has been studied in [23].

where $u_1 = \mathrm{Tr}\,\Phi$ is part of the visible sector, X is the hidden sector and M is a spurion superfield.

There are three appendices in which we collect some useful formulae on elliptic functions, we give a parametric description of the $\mathcal{N}=2$ curve and we provide the details of the solution to the non-holomorphic minimal area equations.

2. Gauge theory analysis: multitrace deformations

We will review $\mathcal{N}=2$ gauge theory broken to $\mathcal{N}=1$ by a superpotential for the adjoint chiral superfield and describe its supersymmetric vacua and its scalar potential. A particular choice of superpotential leads to the existence of local minima of the scalar potential, which are metastable vacua that dynamically break supersymmetry. This has been discussed for $SU(2)$ gauge group in [22] and for generic $SU(N)$ gauge group in [21].

We consider $\mathcal{N}=2$ supersymmetric gauge theories with $U(N)$ gauge group. We will need $U(N)$ rather than $SU(N)$ gauge group because the former is naturally realized by the brane configurations. The chiral ring of the $U(N)$ gauge theory is generated by $u_r = \frac{1}{r}\langle \mathrm{Tr}\Phi^r\rangle$, for $r=1,\ldots,N$, where Φ is the adjoint chiral superfield in the $\mathcal{N}=2$ gauge supermultiplet. If we denote by a_i the classical eigenvalues of the adjoint, then classically we have

$$u_r = \sum_{i=1}^{N} a_i^r \ , \tag{2.1}$$

and the u_r parameterize the moduli space of the Coulomb branch of the $\mathcal{N}=2$ gauge theory, for $r=1,\ldots,N$. At a generic point on the moduli space, the gauge symmetry is broken to its maximal abelian subgroup $U(1)^N$ and the theory is in the Coulomb branch. The chiral ring is conveniently encoded in the characteristic polynomial $P_N(v,u_r) = \det(v-\Phi) = v^N \exp\left(-\sum_{r=1}^{\infty}\frac{u_r}{v^r}\right)$. Since $P_N(v)$ is a degree N polynomial in v, we need to impose that the coefficients of the negative powers in the Laurent expansion vanish. In this way we can express the higher trace operators $u_{r>N}$ in terms of the first N operators $u_{r\leq N}$. The $\mathcal{N}=2$ gauge theory physics is described at low energy by the hyperelliptic curve $y^2 = P_N(v)^2 - 4\Lambda^{4N}$. The expectation values of the chiral ring operators can be read from the curve as

$$u_r = \oint_{\infty} dv \frac{v^r P_N'(v)}{y} \ . \tag{2.2}$$

The first N operators in (2.1) are exact: they do not receive quantum corrections in the $\mathcal{N}=2$ theory, but when $r>N$ they get quantum corrections.

Now we break $\mathcal{N}=2$ supersymmetry to $\mathcal{N}=1$ by adding a tree level superpotential

$$W=\sum_{r=0}^{k} s_r u_{r+1} \ , \tag{2.3}$$

where we will be interested in particular in the case where $k>N$. The higher trace operators u_r are to be understood as multitrace interactions, when written in the usual basis of the first N chiral ring operators.[3] Let us briefly discuss the vacuum structure of these gauge theories. The $\mathcal{N}=2$ theory has a quantum moduli space of supersymmetric vacua parameterized by the u_r for $r=1,\dots,N$. When we add the superpotential (2.3), the moduli space is lifted to a discrete set of supersymmetric vacua, given by the u_r in (2.1) where the eigenvalues of the adjoint are at the roots of

$$W'(v)=s_k\prod_{i=1}^{k}(v-a_i) \ , \tag{2.4}$$

modded out by the Weyl reflection, so the number of classical vacua in the gauge theory is

$$\binom{N+k-1}{N} \ . \tag{2.5}$$

The non-supersymmetric vacua are the non-zero minima of the scalar potential

$$V=g^{i\bar{j}}\partial_i W\,\partial_{\bar{j}}W \ , \tag{2.6}$$

where $g_{i\bar{j}}$ is the Kahler potential of the $\mathcal{N}=1$ gauge theory. In general it is difficult to compute the $\mathcal{N}=1$ Kahler potential, however, in the regime where the superpotential is just a small perturbation, we can reliably use the $\mathcal{N}=2$ Kahler metric on the moduli space

$$g_{r\bar{s}}=\operatorname{Im}\tau_{ij}\frac{da^i}{du_r}\frac{d\bar{a}^j}{d\bar{u}_s} \ , \tag{2.7}$$

where τ_{ij} is the matrix of the low energy $U(1)$ couplings. The authors of [21] showed that any point on the $\mathcal{N}=2$ moduli space of vacua can be lifted to a non-supersymmetric

[3] At low energy in the $\mathcal{N}=2$ theory, the gauge dynamics of the $U(1)$ part is frozen, so the $\mathcal{N}=2$ theory (without the superpotential (2.3)) is effectively $SU(N)$. However, when we add the interaction (2.3), the $U(1)$ part of the adjoint chiral superfield interacts with the remaining $SU(N)$ part through the Yukawa couplings, hence we cannot disregard the $U(1)$ part of the dynamics, that will be crucial to identify the correct vacua in the brane picture.

metastable vacuum by an appropriate choice of superpotential (2.3) with higher trace operators. In particular, if we integrate out the u_1 modulus so that we are left with an $SU(N)$ gauge group, one can lift the origin of the $SU(N)$ moduli space by turning on the tree level superpotential

$$W = \lambda \left(\frac{u_N}{N} + \frac{(N-1)^2}{6N^3} \frac{u_{3N}}{\Lambda^{2N}} \right) \ , \tag{2.8}$$

where u_N, u_{3N} are the $SU(N)$ operators and λ is a small coupling. Moreover, the metastable vacuum at the origin can be made parametrically long lived against decays to both the classical supersymmetric vacua and the quantum vacua at the points where dyons condense, by appropriately tuning the couplings and the dynamical scale.

2.1. Metastable vacua with $U(2)$ gauge group

Let us work out in more detail the case of $U(2)$ gauge group, that will be relevant for the brane configuration. The $\mathcal{N}=2$ chiral ring is generated by u_1 and u_2. If we split the classical $U(2)$ adjoint chiral superfield into its $U(1)$ part and its $SU(2)$ part as $\Phi = \mathbb{1}x + \varphi$, we can express the modulus of the $SU(2)$ gauge group $u = \frac{1}{2}\mathrm{Tr}\varphi^2$ as $u = u_2 - \frac{1}{4}u_1^2$. Therefore, the origin $u = 0$ of the $SU(2)$ moduli space occurs at $u_2 = u_1^2/4$ in the $U(2)$ theory.

Let us first add a tree level mass term for the adjoint, namely $W = mu_2$. In the regime of small mass m we can compute the exact scalar potential, which is simply $V = g^{u_2\bar{u}_2}|m|^2$ with the metric $g_{u_2\bar{u}_2}$ given in (2.7). As explained above, the overall $U(1)$ part does not contribute to the $\mathcal{N}=2$ dynamics, that determines the Kahler potential: the metric for the u_2 modulus is thus same as the metric for the modulus u of the $SU(2)$ gauge theory. Hence, as far as the computation of the scalar potential is concerned, we can integrate out u_1 upon its equations of motion and compute V using the effective superpotential for u_2. The scalar potential in the massive case is depicted in Fig. 1a. It has an extremum at the origin $u_2 = 0$, however it is a saddle point. In Section 4 we will argue why naively one may expect to see this extremum in the brane picture, since it might correspond to a solution to the M theory equations of motion. However, the actual M theory computation will show that there is no such solution at all.

We would like to study the metastable supersymmetry breaking vacuum found in [21][22]. Let us introduce the superpotential

$$W = s_1 u_2 + s_5 u_6 \ , \tag{2.9}$$

whose equations of motion can be written as

$$\begin{aligned} u_1 u_2(2u_2 - u_1^2) &= 0 \ , \\ s_1 + s_5\left(u_2^2 + 2u_1^2 u_2 - \frac{1}{4}u_1^4\right) &= 0 \ , \end{aligned} \tag{2.10}$$

where we expressed u_6 in terms of the u_1 and u_2.

We have six solutions for u_1 to be integrated out, giving an effective potential for u_2

$$\begin{aligned} u_1 = 0 \qquad &\Rightarrow W_{eff}^{(1)} = s_1 u_2 + \frac{s_5}{3}u_2^3 \ , \\ u_1 = 2u_2 \qquad &\Rightarrow W_{eff}^{(2)} = s_1 u_2 + \frac{4s_5}{3}u_2^3 \ , \\ u_1^2 = \pm\sqrt{4s_1/s_5} \qquad &\Rightarrow W_{eff}^{(3)} = \pm - 2\sqrt{s_1 s_5}u_2^2 + \frac{s_5}{3}u_2^3 \ . \end{aligned} \tag{2.11}$$

The scalar potential $V^{(i)}(u_2) = g^{u_2\bar{u}_2}|\partial_{u_2}W_{eff}^{(i)}|^2$ will have three different expressions on the three different branches in (2.11). The analysis in each branch reduces then to the one in [21][22] and it turns out that $V^{(1)}$ and $V^{(2)}$ display a metastable vacuum at the origin of the u_2 moduli space in a special range of the coupling s_5/s_1, $\lambda_-^{(i)} < \frac{s_1}{s_5} < \lambda_+^{(i)}$ where $\lambda_\pm^{(1)} = 1/24 \pm \left(\frac{\Gamma(3/4)}{2\Gamma(5/4)}\right)^4$ and $\lambda_\pm^{(2)} = 4\lambda_\pm^{(1)}$. The metastable vacuum is shown in Fig. 1b.

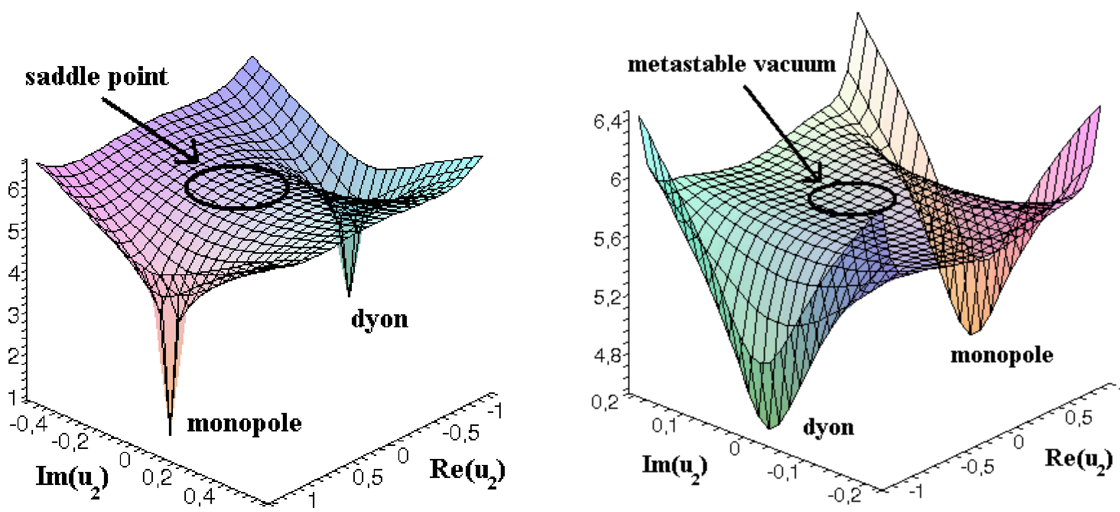

Fig. 1: Plot of the scalar potential. In Fig.1a, the superpotential $W = mu_2$ gives a saddle point at the origin. In Fig.1b, the superpotential $W_{eff}^{(1)}$ in (2.11) transforms the saddle point into a local minimum.

Let us comment on the physics of the classical supersymmetric vacua (2.10). They correspond to generic values of the moduli u_1 and u_2. Close to the origin of the moduli space, once we integrate out the dynamics of the abelian factor corresponding to u_1, the softly broken $U(2)$ gauge theory admits an effective description in terms of an abelian gauge theory coupled to two chiral superfields M and $\widetilde{M}$, representing magnetic monopoles, whose superpotential is

$$\widetilde{W} = \widetilde{M} A M + \sum_{i=1}^{k} s_i u_i \ , \tag{2.12}$$

where the last term is (2.11). At a generic point on the moduli space, the equations of motion of (2.12) set $\widetilde{M} = M = 0$: the monopoles are massive, so the curve is not degenerate. In addition, there are two extra supersymmetric vacua where a monopole or a dyon condenses and the curve degenerates.

The reason for the existence of these metastable vacua is the following [21]. The metric (2.7) on the moduli space of the $\mathcal{N} = 2$ Coulomb branch has positive definite curvature almost everywhere. There exists therefore a suitable superpotential such that any point on the $\mathcal{N} = 2$ moduli space can be lifted to a metastable vacuum. Around any regular point on the moduli space, one can go to the coordinate system z^i, for $i = 1, \dots, N$, adapted to that point. Then, it is generically possible to choose a superpotential cubic in z^i, such that the scalar potential (2.6) has a local minimum at the origin, in this coordinate system. Higher powers than cubic in general do not affect the metastability of the vacuum, and in fact one can add such irrelevant terms as long as their couplings are small. The analysis is valid when the superpotential is treated as a small perturbation, so that one can trust the $\mathcal{N} = 2$ Kahler metric.

3. Supersymmetric vacua in the brane picture: multitrace deformations

We have discussed how the gauge theory with the multitrace superpotential (2.8) develops a metastable vacuum at the origin of the $\mathcal{N} = 2$ moduli space. On top of that, the multitrace deformation (2.8) gives rise to a large number of supersymmetric vacua. In this Section we will discuss the type IIA description of these gauge theory supersymmetric vacua and their lift to M theory. Due to the fact that the degree of the superpotential is larger than the number of color, the M theory lift will be different from the ones studied in the past (for a review see [3] and references therein), where the degree of the superpotential was taken to be at most equal to the rank of the gauge group. The new ingredient is that the "excess" $k-N$ NS fivebranes, once lifted to M theory, become a bunch of disconnected components of the M5-brane worldvolume.

3.1. Type IIA setup

Let us consider type IIA string theory in flat ten dimensions. The brane configuration describing the classical $\mathcal{N}=1$ gauge theory with degree $k+1$ superpotential (2.3) consists of one fivebrane NS, k fivebranes NS' and N D4-branes, whose worldvolumes extend along

$$\begin{array}{ccccccccc} & x_0 & x_1 & x_2 & x_3 & v & x_6 & x_7 & w \\ NS & \bullet & \bullet & \bullet & \bullet & \bullet & \times & \times & \times \\ NS' & \bullet & \bullet & \bullet & \bullet & / & \times & \times & / \\ D4 & \bullet & \bullet & \bullet & \bullet & \times & \bullet & \times & \times \end{array} \tag{3.1}$$

where $v=x_4+ix_5$ and $w=x_8+ix_9$. The k NS' branes are rotated in the (v,w) directions and stuck at a point in x_6. The gauge theory eigenvalues of the $U(N)$ adjoint Φ correspond in the brane picture to the positions of the D4-branes along the $v=x_4+ix_5$ direction at $w=0$. In particular, the $U(1)$ part $u_1/N=\mathrm{Tr}\Phi/N$ of the adjoint represents the center of mass coordinate of the system of the D4-branes, while the operators u_r for $r=2,\ldots,N$ parameterize the relative displacement of the D4-branes in the v-plane. When the k NS' are rotated in the (v,w) direction, their position along the v direction at $w=0$ is given by the solutions of the classical equations of motion in which all the D4-branes are on top of each other, namely for $u_{r>1}=0$ and $u_1\neq 0$. In particular, given a generic superpotential $W(\Phi)$ in (2.4), the k NS' branes intersect the plane $w=0$ at $v=a_i$. The number of ways to suspend the $N<k$ D4-branes between the NS and the k NS' is precisely (2.5), showing the one to one correspondence with the classical supersymmetric vacua of the gauge theory.

As an illustrative example, consider the classical $U(2)$ gauge theory superpotential (2.9) responsible for the metastable vacua in Fig. 1b. The positions of the $k=5$ NS' branes are determined by $W^{(1)}_{eff}$ in (2.11), and we have drawn their locations in the v plane in Fig. 3. Note that in order to reproduce correctly the vacua it is crucial to take into account the Yukawa couplings between the $U(1)$ modulus u_1 and the nonabelian part of the adjoint superfield.

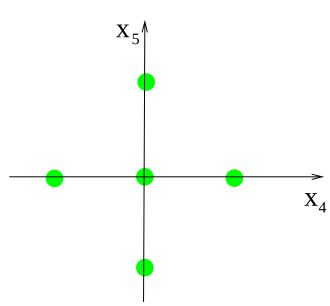

Fig. 2: The positions of the $k=5$ NS' branes in the $v=x_4+ix_5$ plane at $w=0$, corresponding to the gauge theory vacuum $W^{(1)}_{eff}$ (2.11).

3.2. M theory lift and disconnected curves

We want to discuss the lift to M theory of the classical gauge theory vacua, which are in one to one correspondence to the classical brane configurations. The $\mathcal{N} = 2$ supersymmetric theory has a moduli space of vacua. It corresponds to parallel NS and NS' branes, extended along the v-plane at $w = 0$. In this case the D4-branes are free to move in the v-direction, and their positions parameterize the Coulomb branch of the gauge theory. When we add an $\mathcal{N} = 1$ superpotential to the gauge theory, the moduli space is lifted, leaving just an isolated number of vacua. We have N isolated vacua, corresponding to the points at which a massless monopole condenses, that in the low energy theory represent the N gaugino condensate vacua. In addition to that, we have more supersymmetric vacua, given by the solution to the F-term equations.

Consider the vacuum in which each of the N D4-branes is attached to a different NS' brane. The k NS' branes are located at the roots of (2.4). We separate them in two sets: to the first N of them, that intersect $w = 0$ at the positions $v = a_1, \ldots, a_N$, we attach the N D4-branes; the remaining $k - N$ NS' are just spectators, and we place them at the positions $v = a_{N+1}, \ldots, a_k$. When we switch on the type IIA string coupling $g_s << 1$, the eleventh dimensional circle x_{10} opens up. As usual we introduce a new complex coordinate as $t = \exp\left[-(x_6 + ix_{10})/R\right]$, where R is the M theory radius. Quantum mechanically, a D4-brane ending on the NS5 brane bends it at infinity. The classical brane configuration we have just described consists then of three different asymptotic regions in M theory as shown in Fig. 3. The first region is at $x_6 = -\infty$, that is $t = \infty$, where the NS brane is bent by all the N D4-branes attached to it

$$t \sim \infty, \quad v \sim \infty : \qquad \begin{array}{c} t \sim 2v^N \\ w \sim s_k \Lambda^{2N}/v^N \end{array} . \tag{3.2}$$

The second asymptotic region is at $x_6 = \infty$, that is $t = 0$, where we have N NS' branes, which are bent by the N D4-branes attached to them. Each NS' is rotated in the (v, w) plane, so that at infinity we need N different solutions for w as a function of v

$$t \sim 0, \quad v \sim \infty : \qquad \begin{array}{c} t \sim 2\Lambda^{2N}/v^N \\ w \sim s_k \prod_{i=1}^N (v - a_i) \end{array} . \tag{3.3}$$

The third asymptotic region corresponds to the $k - N$ spectator NS' branes, to which no D4-brane is attached. Since they feel no force, their worldvolume is flat and extend at an angle in the (v, w) direction and at a fixed position $t = t_0$

$$t \sim t_0, \quad v \sim \infty : \qquad w \sim s_k \prod_{i=N+1}^{k} (v - a_i). \tag{3.4}$$

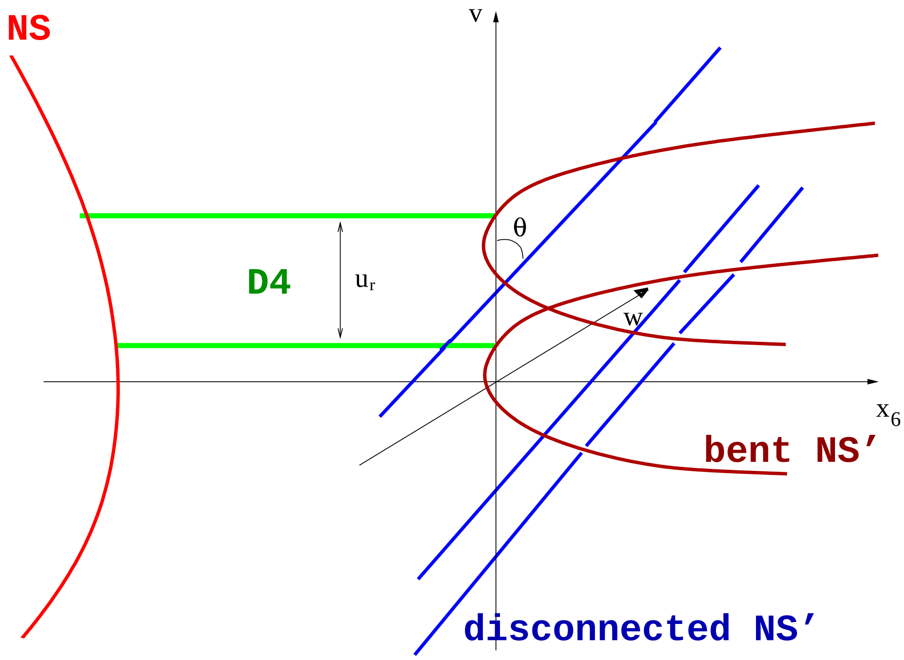

Fig. 3: The small g_s description of the $U(2)$ gauge theory with $k=5$. The NS' branes are at an angle θ in the (v,w) plane. The brane configuration corresponds to a vacuum with a nonzero vev for both u_1 and u_2 moduli. The asymptotic region (3.2) corresponds to the red NS brane on the right; the asymptotic region (3.3) corresponds to the brown NS' branes on the right; the third asymptotic region (3.4) corresponds to the flat blue NS' branes. The green lines are the two D4-branes suspended between the NS and the NS' fivebranes.

The M theory configuration that satisfies these three asymptotic boundary conditions is a fivebrane with worldvolume $R^{1,3}\times\Sigma$, where

$$\Sigma=\Sigma_c\cup\Sigma_d\ , \tag{3.5}$$

is a holomorphic curve consisting of two disconnected components. In the case in which each D4 brane ends on a different NS' brane, the component Σ_c of the fivebrane satisfying the first and second boundary conditions (3.2)-(3.3) is given by

$$\Sigma_c:\qquad\left\{\begin{array}{c} v=\left(\frac{t}{2}\right)^{\frac{1}{N}}+\Lambda^2\left(\frac{2}{t}\right)^{\frac{1}{N}}\ , \\ w=s_k\prod_{i=1}^{N}(v-a_i)\ . \end{array}\right. \tag{3.6}$$

The first equation is the usual Seiberg-Witten curve for the $U(N)$ gauge theory at the point in the moduli space in which it degenerates to a sphere. The second disconnected component Σ_d of the fivebrane worldvolume simply consists of the collection of the spectator flat

$k - N$ NS' branes and is given by[4]

$$\Sigma_d : \qquad \begin{cases} t = t_0 \ , \\ w = s_k \prod_{i=N+1}^{k} (v - a_i) \ , \end{cases} \tag{3.7}$$

4. M5-brane non-supersymmetric vacua: no metastable spontaneous breaking

In this Section we will discuss the issue of metastable spontaneous supersymmetry breaking in the framework of an M5-brane wrapping a non-holomorphic curve.

Recall that in gauge theory a metastable supersymmetry breaking vacuum is realized as follows: one introduces a supersymmetric lagrangian and computes the scalar potential (2.6). A local minimum with non-zero energy breaks supersymmetry spontaneously. If there are other minima at lower energies, then the supersymmetry breaking minimum is metastable towards tunnelling to these other vacua, and in order to be phenomenologically interesting, it must be long-lived, i.e. its decay to the other lower energy minima being parametrically small.

We may translate this discussion to the M5-brane framework, by defining a spontaneous breaking of supersymmetry as a wrapping of a non-holomorphic minimal volume curve with holomorphic boundary conditions at infinity. These holomorphic boundary conditions correspond to the holomorphic classical superpotential in the gauge theory. A stable non-supersymmetric minimum such as the IYIT model [26], realized on the branes in [6], will translate to having no holomorphic curve and only a non-holomorphic minimal volume curve with holomorphic boundary conditions. On the other hand, having both a supersymmetric vacuum and a non-supersymmetric one corresponds to two different solutions to the minimal volume equations with same holomorphic boundary conditions: one holomorphic curve, corresponding to the supersymmetric vacuum, and one non-holomorphic curve, corresponding to the metastable non-supersymmetric vacuum.

In the case of the metastable vacuum found in $\mathcal{N} = 1$ SQCD with massive flavors [1], it has been shown that the M5-brane theory does not realize the gauge theory metastable vacuum [7]. As we noted before, this is not unexpected, since the M5-brane framework regime of validity and the gauge theory one are not the same. Indeed, here as well we will see that the metastable vacuum in softly broken $\mathcal{N} = 2$ gauge theory [21][22] are not realized on the worldvolume of the M5-brane. Once we fix holomorphic boundary conditions at infinity, we find only holomorphic minimal volume curves.

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[4] The general case, in which multiple D4 branes end on each NS' branes, is given by a partial degeneration of the $\mathcal{N} = 2$ curve. It is discussed in Eq. (4.21) of [25]. The disconnected part Σ_c can be easily obtained as well, as a collection of the leftover disconnected NS' branes.

4.1. The minimal volume equations

The worldvolume of the M5-brane is $R^{1,3} \times \Sigma$, where Σ is a two-dimensional curve. If we consider a Nambu-Goto form for the bosonic part of the action, then the area of the two- dimensional curve plays the role of the potential energy [5]

$$\mathrm{Area}(\Sigma_g) = \int_{\Sigma_g} d^2x \sqrt{g} \ , \tag{4.1}$$

where g is the determinant of the induced metric on the worldvolume. The area element can be expressed as

$$\sqrt{g} d^2x = g_{z\bar{z}} d^2z = G_{i\bar{j}} \left(\partial_z X^i \partial_{\bar{z}} X^{\bar{j}} + \partial_{\bar{z}} X^i \partial_z X^{\bar{j}} \right) d^2z \ , \tag{4.2}$$

where $G_{i\bar{j}}$ is the spacetime metric. The equations of motion are then equivalent to solving for a minimal area surface, namely the embedding coordinates X^i and $X^{\bar{j}}$ must satisfy

$$G_{i\bar{j}} \partial_{\bar{z}} \partial_z X^i + \partial_{\bar{z}} X^i \partial_z X^k \frac{\partial}{\partial X^k} G_{i\bar{j}} = 0 \ , \tag{4.3}$$

and the Virasoro constraint

$$G_{i\bar{j}} \partial_z X^i \partial_z X^{\bar{j}} = 0 \ . \tag{4.4}$$

In our setup, the spacetime embedding coordinates are $X^i = (w, v, s), X^{\bar{i}} = (\bar{w}, \bar{v}, \bar{s})$. When the spacetime metric is flat, the second term in (4.3) drops.

We would like to find embedding coordinates (w, v, s), which satisfy the M5-brane equations of motion and Virasoro constraints. The first condition is

$$\partial\bar{\partial} s = \partial\bar{\partial} v = \partial\bar{\partial} w = 0 \ , \tag{4.5}$$

which is solved by harmonic functions

$$\begin{aligned} s(z, \bar{z}) =& s_H(z) + \overline{s_A(z)} \ , \\ v(z, \bar{z}) =& v_H(z) + \overline{v_A(z)} \ , \\ w(z, \bar{z}) =& w_H(z) + \overline{w_A(z)} \ . \end{aligned} \tag{4.6}$$

The Virasoro constraint (4.4) reads

$$g_s^2 \partial s_H \partial s_A + \partial v_H \partial v_A + \partial w_H \partial w_A = 0 \ , \tag{4.7}$$

where the g_s factor comes from the metric. Note that a holomorphic curve automatically satisfies both equations of motion (4.5) and (4.6).

[5] Since the curves are non-compact this area is infinite, it needs to be regularized, as we discuss later on.

4.2. Breaking $\mathcal{N}=2$ to $\mathcal{N}=1$ in the brane picture

Let us first recall the parametric description of the $\mathcal{N}=2$ holomorphic curve for $U(2)$ gauge theory in terms of a torus with coordinate z and period τ as in Fig.4. More details are given in the Appendix B. The embedding coordinates at a point where $U(2)$ is broken to $U(1)\times U(1)$ are given by

$$\begin{aligned} s_{SW}(z) =& 2(F(z-a_1)-F(z-a_2)-\pi i z) \ , \\ v_{SW}(z) =& A(F^{(1)}(z-a_1)-F^{(1)}(z-a_1)-i\pi)+\frac{1}{2}u_1 \ , \\ w_{SW}(z) =& 0 \ , \end{aligned} \tag{4.8}$$

where the relation between A,τ and the usual moduli and dynamical scale are derived in Appendix B. We have introduced the function $F(z)=\ln\theta_3[\pi(z-\tilde{\tau})]$, where $\tilde{\tau}=(\tau+1)/2$, and denoted its derivative by $F^{(1)}(z)$. The properties of this function are discussed in Appendix A, following the conventions in [19]. The embedding coordinates satisfy the following boundary conditions at the NS and NS' branes

$$NS:\ z\sim a_2 \quad \left\{\begin{array}{c} w\sim 0\ , \\ v\sim\infty\ , \\ t\sim 2v^N\ . \end{array}\right. \qquad NS':\ z\sim a_1 \quad \left\{\begin{array}{c} w\sim 0\ , \\ v\sim\infty\ , \\ t\sim 2\Lambda^{2N}/v^N\ , \end{array}\right. \tag{4.9}$$

where $t=e^{-s}$. In the formula (4.9) and in the following we understand that $N=2$ and $N_1=N_2=1$, but we sometimes keep the number of colors explicit.

When studying $\mathcal{N}=1$ holomorphic curves, we parameterized the boundary conditions for the embedding coordinates $t=e^{-s}$ as in (3.2) and (3.3), which in the $\mathcal{N}=2$ case reduce to (4.9). It is convenient to recast these asymptotics in another form, which will be more appropriate when studying non-holomorphic curves [19]. These definitions are valid for $\mathcal{N}=2,1,0$. We specify the periods and residues of the differential ds on the torus: the residue at the location of the NS brane is the total number of $D4$-branes, equal to the rank N of the gauge group

$$Res_{a_1}ds = Res_{a_2}ds = 2\pi i N \ . \tag{4.10}$$

The A-periods in the $\mathcal{N}=1$ language correspond to the ranks of the low energy gauge groups

$$\oint_{A_i} ds = 2\pi i N_i \ , \tag{4.11}$$

which in the type IIA picture represents the number of D4-branes that are piled up in the same stack, which on the M theory side it is the number of times the M5-brane wraps

the eleventh dimensional circle. In our case $N_i = 1$, since we are on the $\mathcal{N} = 2$ Coulomb branch (the A_2-period is defined up to the residue at a_1). Then we have the constraint that the total B period is an integer. The compact B periods are the differences in the theta angle of consecutive low gauge groups, but in our case we just have abelian gauge groups, so we fix it to zero

$$\oint_{B_1 - B_2} ds = 0. \tag{4.12}$$

If we introduce a cutoff v_0 for v, when z is close to the marked points a_1, a_2, then the B-periods of ds give the four-dimensional running gauge couplings at the scale v_0

$$\oint_{B_i} ds = 2\pi i \alpha_i(v_0) \ , \tag{4.13}$$

where $\alpha(v_0) = \frac{\theta}{2\pi} + \frac{4\pi i}{g^2_{YM}(v_0)}$. In the $\mathcal{N} = 2$ case (4.13) reproduces the one loop part of the exact τ

$$\oint_{B_2} ds = -2 \ln \frac{v_0^2}{\Lambda^2} \tag{4.14}$$

where Λ is the $\mathcal{N} = 2$ dynamical scale. In the holomorphic case, fixing these periods of ds gives back the boundary conditions (4.9). The solution (4.8) satisfies the various boundary conditions (4.10), (4.11). The constraint (4.12) requires

$$a \equiv a_2 - a_1 = -\frac{\tau}{2} \ . \tag{4.15}$$

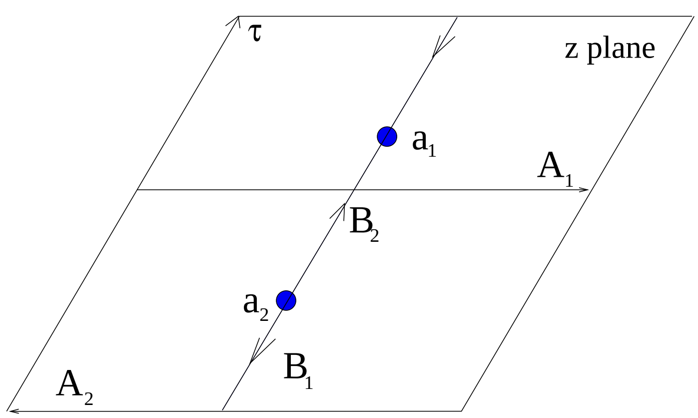

Fig. 4: The parametric description of the torus in the z plane. The marked points at $z = a_1, a_2$ are the location of the NS' and NS fivebranes. Their distance is fixed to $a_2 - a_1 = -\tau/2$. The cycles B_1 and B_2 are non-compact.

Let us break now the $\mathcal{N}=2$ supersymmetry to $\mathcal{N}=1$ by the superpotential

$$W = mu_2 = \frac{m}{2}\mathrm{Tr}\Phi^2 \ . \tag{4.16}$$

The corresponding boundary conditions (4.16) are usually taken to be (4.9) at the NS brane, namely $w=0$, while at the NS' brane one takes $w = mv$. For later convenience, we perform a rotation in the (w,v) space and take more symmetric boundary conditions given by

$$NS: \ z \sim a_2 \qquad \begin{cases} w \sim -mv \ , \\ v \sim \infty \ , \\ t \sim 2\Lambda^4/v^2 \ . \end{cases} \qquad NS': \ z \sim a_1 \qquad \begin{cases} w \sim mv \ , \\ v \sim \infty \ , \\ t \sim 2v^2. \end{cases} \tag{4.17}$$

As expected from the gauge theory, there is no holomorphic torus with holomorphic boundary conditions (4.17). In fact, w would be an elliptic (i.e. doubly periodic) meromorphic function with non-zero residue and this is not possible. Note that if, instead, we look for a holomorphic curve with these boundary conditions but with genus zero, then we find the holomorphic lift of the $\mathcal{N}=2$ monopole and dyon points, where the torus degenerates to a sphere [24]. The latter are indeed supersymmetric vacua.

4.3. M5-brane: no metastable spontaneous supersymmetry breaking

If we introduce the superpotential (4.16), then the gauge theory scalar potential $V(u_2)$, computed in the approximation of small mass m, has an extremum (saddle point) at the origin $u_2 = 0$ as we showed in Fig. 1a. In the following we ask whether there is in the M5-brane framework a corresponding non-holomorphic curve with asymptotically holomorphic boundary conditions [6].

We will first look for a non-holomorphic minimal area torus with boundary conditions (4.17), corresponding to the massive gauge theory (4.16). Then, in the next Section we will consider the multitrace deformation (2.9), that on the gauge theory side gives rise to a metastable vacuum. As we have discussed in Section 3, this deformation is realized in the brane picture by adding disconnected parts of the M5-brane worldvolume. We will attempt to take the change in the brane configuration into account by considering the effect of the gravitational interaction of the disconnected curves on the part of the curve that in the type IIA limit contains the D4-branes. Our analysis will show that the

[6] In order to distinguish a saddle point from a minimum one may study the spectrum of fluctuations around the solution.

gravitational interaction of the disconnected components can be actually neglected and we are back to the first case. We will see that the M5-brane does not exhibit the metastable non-supersymmetric vacua.

Let us see what goes wrong if we try to lift to M theory the type IIA intersecting brane configuration at the origin of the moduli space. At $u_2 = 0$, the gauge symmetry in the quantum theory is broken to $U(1) \times U(1)$, hence would-be the curve is a torus, parameterized by the holomorphic coordinate z. The position of each NS5 brane at infinity is a marked point on the torus at $z = a_1, a_2$. This means that the surface is non-compact.

If we try to lift (4.17) with a non-holomorphic curve, we encounter a problem. The equations of motion (4.6) imply that w, v and s are harmonic and elliptic functions. We can achieve this by adding to (4.8) and appropriate anti-holomorphic part such that, in particular $w(z,\bar{z}) \sim \pm m v(z,\bar{z})$ at $z \to a_{1,2}$, and the functions are elliptic. However, we have to satisfy the Virasoro condition (4.7) and it is easy to see that it is not possible to find such elliptic functions, not even at first order in the small mass parameter m. This is because, when we try to satisfy (4.7) in the vicinity of the marked points $z \sim a_1, a_2$, the contribution from w and v contain a fourth order pole, whose coefficient is always proportional to $1 + |m|^2$, that never vanishes. On the other hand, a higher trace superpotential e.g. such as (2.9), corresponds to boundary conditions of the form $w \sim v^k + \ldots$. This case is even worse than the previous one. In fact, although it is possible to have harmonic elliptic functions with these boundary conditions, the leading contribution of w to the Virasoro condition (4.7) is now a pole of degree $2k+2$, while the leading contribution of v is still of fourth order and they do not cancel.

4.4. Backreaction of the disconnected components of the fivebrane

Let us introduce the higher trace deformation (2.9) and incorporate the gravitational interaction of the disconnected components of the M5-brane. The eleven dimensional metric $G_{i\bar{j}}$ is sourced by the $k-1$ parallel fivebranes, rotated in the (v, w) directions by an angle θ, each of which intersect the v-plane at $v = v_i$

$$ds^2_{11} = f^{-\frac{1}{3}} dx^2_{\parallel} + f^{\frac{2}{3}} (dr^2_{\perp} + r^2_{\perp} d\Omega^2_4) \ , \tag{4.18}$$

$$f = 1 + \sum_{i=1}^{k-1} \frac{c}{|r - r_i|^3}, \tag{4.19}$$

and the transverse coordinate reads

$$|r-r_i|_\perp^2 = x_7^2+|s|^2+\cos^2\theta|w|^2+\sin^2\theta(v-v_i)(\bar{v}-\bar{v}_i)-\sin\theta\cos\theta(\bar{w}(v-v_i)+w(\bar{v}-\bar{v}_i)) \ . \tag{4.20}$$

The metric $G_{i\bar{j}}$ has the following non-zero components in the (v,w,s) directions

$$\begin{aligned} G_{v\bar{v}} =& \cos^2\theta f^{-\frac{1}{3}}+\sin^2\theta f^{\frac{2}{3}} \simeq 1+\left(-\frac{1}{3}+\sin^2\theta\right)\sum_i \frac{c}{|r-r_i|_\perp^3} \ , \\ G_{w\bar{w}} =& \sin^2\theta f^{-\frac{1}{3}}+\cos^2\theta f^{\frac{2}{3}} \simeq 1+\left(-\frac{1}{3}+\cos^2\theta\right)\sum_i \frac{c}{|r-r_i|_\perp^3} \ , \\ G_{s\bar{s}} =& f^{\frac{2}{3}} \simeq 1+\frac{2}{3}\sum_i \frac{c}{|r-r_i|_\perp^3} \ , \\ G_{v\bar{w}} =& \sin\theta\cos\theta(f^{-\frac{1}{3}}-f^{\frac{2}{3}}) \simeq -\sin\theta\cos\theta\sum_i \frac{c}{|r-r_i|_\perp^3} \ , \end{aligned} \tag{4.21}$$

where we expanded at large distance r from the source NS' branes. In order for the target space metric to be hermitian we need to require $\theta=\bar{\theta}$.

Since we are looking at the solution to the equations of motion and the Virasoro constraint (4.3) and (4.4) in terms of elliptic functions, we just need to evaluate these equations around one pole, say $z\sim a_1$. The marked points on the torus represent the location of the bent NS and NS' around infinity, where the distance between the two bent fivebranes and the source spectator fivebranes is very large. Hence, the metric there is flat to leading order, so the leading terms in (4.3) and (4.4) should be equal to the flat space ones (4.5) and (4.7). Let us see how this works.

For the equations of motion (4.3) we need the variation of the metric (4.21). If we evaluate (4.3) around the location of the bent NS' at $z\sim a_1$, and then plug the boundary condition $w=\tan\theta\, v$, the v and w equations of motion become equal and read

$$\left[1-\frac{1}{3}\sum\frac{1}{r^3}\right]\partial_z\bar{\partial}_{\bar{z}}\bar{v}+\frac{1}{2}s\bar{\partial}_{\bar{z}}\bar{s}\partial_z\bar{v}\sum\frac{1}{r^5}=0 \ . \tag{4.22}$$

where we introduced the short-hand notation

$$\sum_{i=1}^{k-1}\frac{c}{|r-r_i|_\perp^\alpha}\equiv\sum\frac{1}{r^\alpha}.$$

For s we get an analogous expression. On the other hand, evaluating the Virasoro condition (4.4) around $z\sim a_1$ and using the metric (4.21), once again we can plug directly the boundary condition $w=\tan\theta\, v$ and we get

$$\left[\left(1-\frac{1}{3}\sum\frac{1}{r^3}\right)(1+\tan^2\theta)+\sin^2\theta\sum\frac{1}{r^3}\right]\partial_z v\partial_z\bar{v}+\left[1+\frac{2}{3}\sum\frac{1}{r^3}\right]\partial_z s\partial_z\bar{s}=0 \ , \tag{4.23}$$

which is now an equation for the asymptotic behavior of v, s around $z \sim a_1$.

Finally, one has to solve (4.22) and (4.23) for the embedding functions v, w, s around the pole at $z \sim a_1$. Note that the leading terms in the radius expansion are just the flat space equations (4.5) and (4.7). But we have already seen that there is no non-holomorphic solution to these equations. Hence, we conclude that the M5-brane framework does not exhibit the metastable gauge theory vacuum, even when including the backreaction of the spectator components of the fivebrane.

5. M5-brane non-supersymmetric vacua: soft breaking

In this Section we will consider M5-branes wrapping minimal volume non-holomorphic curves with non-holomorphic boundary conditions. We will later interpret this type of supersymmetry breaking as the analog of soft supersymmetry breaking in gauge theory. While in the case of a holomorphic rotation $w = mv$ only the monopole and dyon points are lifted to $\mathcal{N} = 1$ vacua, we will see that with the non-holomorphic rotation $w = m(v+\bar{v})$ any point on the $\mathcal{N} = 2$ moduli space is lifted to a non-supersymmetric vacuum.

5.1. A non-holomorphic torus

We have seen that the M5-brane does not exhibit the analog of metastable gauge theory vacua. In particular, it is not possible to realize the holomorphic boundary conditions (4.17) with a minimal volume non-holomorphic curve. In the following we will look for a non-holomorphic M5-brane configuration which is as close as possible to the one in (4.17). We will take the boundary conditions to be non-holomorphic but still with a *linear* relation between w and v. As we will discuss, this may describe the analog of soft supersymmetry breaking in the gauge theory.

We look for a minimal area curve that satisfies the following conditions:
i) It is a genus one curve with *non-holomorphic* embeddings in the target space coordinates (w, v, s). The embeddings have to be harmonic functions of z as in (4.6).
ii) We consider a rotation with a mass parameter m, such that, when we take m to zero, we recover the $\mathcal{N} = 2$ curve (4.8). Thus, our exact solution can be considered for small m as a perturbation of the $\mathcal{N} = 2$ theory.
iii) We require that at infinity, at first order in m and in g_s, the rotation is of the form

$$w \sim (v + \bar{v}) \ . \tag{5.1}$$

Note, that if w is proportional to a higher power of v, it is hard to solve the Virasoro constraint, since the higher order poles coming from the w contribution cannot be cancelled. *iv*) We fix the periods and residues of ds as in (4.10), (4.11), (4.12), (4.13).

The requirements that w diverges linearly at the NS and the NS' branes means that it can only depend on $F^{(1)}(z-a_i)$ and its complex conjugate, but not on higher derivatives $F^{(k>1)}$, since $F^{(n)}(z-a_i)$ has an n-th order pole at $z=a_i$. Moreover, requiring that it is harmonic and elliptic fixes its form uniquely to

$$w = mg_s A\left(F^{(1)}(z-a_1)+F^{(1)}(z-a_2)+\overline{F^{(1)}(z-a_1)}+\overline{F^{(1)}(z-a_2)}\right) \ , \tag{5.2}$$

where the factor of g_s will be necessary to satisfy the Virasoro constraint (4.7). The embedding function v can be parameterized as

$$\begin{aligned} v =& A\left(F^{(1)}(z-a_1)-F^{(1)}(z-a_2)-i\pi\right)+\frac{1}{2}u_1 - Ag_s^2m^2\left(\overline{F^{(1)}(z-a_1)}-\overline{F^{(1)}(z-a_2)}\right) \\ &+m^2g_s^2\rho\left(F^{(1)}(z-a_2)+\overline{F^{(1)}(z-a_2)}\right) \ , \end{aligned} \tag{5.3}$$

where the first term is the SW solution (4.8) and we fix the overall scale $A(\tau)$ by consistency with the $m=0$ limit. Finally, the embedding function $s(z,\bar{z})$ acquires an anti-holomorphic part as well

$$\begin{aligned} s =& 2(F(z-a_1)-F(z-a_2)-i\pi z) \\ &+m^2\gamma\left(F(z-a_1)-F(z-a_2)+\overline{F(z-a_1)}-\overline{F(z-a_2)}-i\pi(z-\bar{z})\right) \ , \end{aligned} \tag{5.4}$$

and note in particular that its holomorphic and anti-holomorphic parts are

$$s_H = \left(1+\frac{m^2\gamma}{2}\right)s_{SW}(z) \ , \qquad s_A = \frac{m^2\gamma}{2}s_{SW}(z) \ . \tag{5.5}$$

One can easily check that this satisfies our period conditions (4.11) and (4.12) that fix the distance between the two marked points on the torus, namely $a \equiv a_2 - a_1 = -\tau/2$, as in the $\mathcal{N}=2$ case (see appendix B). The last boundary condition (4.13) fixes the dependence of the period τ of the torus boundary data at the cutoff scale

$$\begin{aligned} 2\pi i\alpha(v_0) =& -2\left(\ln\left(\frac{v_0}{A}\right)^2+2F(\tau/2)-2\ln\theta'(\tilde{\tau})-i\pi\tau/2\right) \\ &-m^2\gamma\left(\ln\left(\frac{v_0}{A}\right)^2+2F(\tau/2)-2\ln\theta'(\tilde{\tau})-i\pi\tau/2+c.c.\right) \end{aligned} \tag{5.6}$$

Since we have fixed $A(\tau)$ by the $\mathcal{N}=2$ limit, (5.5) takes a very simple form

$$2\pi i\alpha(v_0) = -2\ln\left(\frac{v_0}{\Lambda}\right)^2 - m^2\gamma\left|\frac{v_0}{\Lambda}\right|^4 \ . \tag{5.7}$$

Here and in the following we still will denote by Λ the scale of the unperturbed $\mathcal{N}=2$ theory.

The parameters ρ,γ in (5.3) and (5.4) are fixed by the Virasoro condition (4.7). As anticipated above, the way to satisfy this constraint is the following. The functions appearing in the constraint are all elliptic functions. Hence, we just need to expand them around a pole, say $z=a_1$, and require that the coefficients of the poles of different degrees and the constant term in the expression vanish. This fixes the various coefficients in the embedding functions. The fourth order pole has already been cancelled by the $m^2 g_s^2$ term in (5.3), so we are left with a double pole, a single pole and a constant, each of which must vanish separately. The details of this computation are given in appendix C. The result is that ρ,γ depend on τ, m, g_s as follows

$$\begin{aligned}\gamma =& -\frac{1}{m^2}+\frac{1}{m^2}\left[\frac{\alpha+(1-m^2g_s^2)A^2\beta^{\frac{1}{2}}}{8g_s^2(\wp+2\eta_1)}\right]^{\frac{1}{2}} \ ,\\ \rho =& \frac{1}{m^2g_s^2-1}\left(4A+\frac{2\gamma+m\gamma^2}{Am(\wp+2\eta_1)}\right) \ ,\end{aligned} \tag{5.8}$$

where $\alpha=\alpha(\tau,g_s,m)$, $\beta=\beta(\tau,g_s,m)$, the Weierstrass function $\wp(\tau/2)$ and $\eta_1(\tau)$ are certain elliptic functions defined in Appendix C. We can look at their leading order expansion as we take $m\to 0$ to check that we get back the $\mathcal{N}=2$ solution of (4.8) in this limit. Both γ and ρ are indeed finite in this limit. In Fig.5 and 6 we plot the two coefficients as functions of τ imaginary.

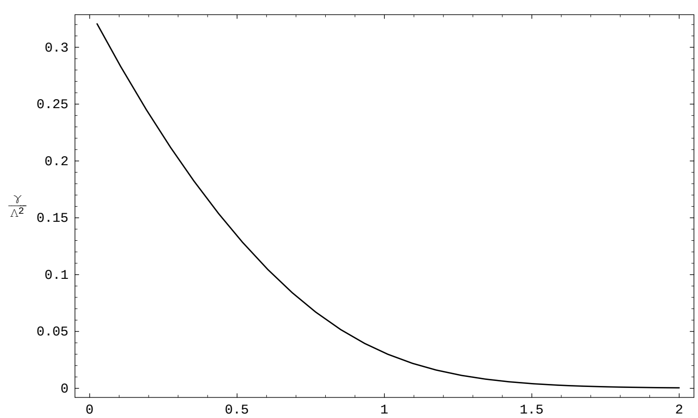

Fig. 5: Plot of the coefficient $\gamma(\tau)$ in (5.8) for imaginary values of τ and small m. It is monotonic for any m and intercepts $\gamma(\tau=0)=\Lambda^2/3$.

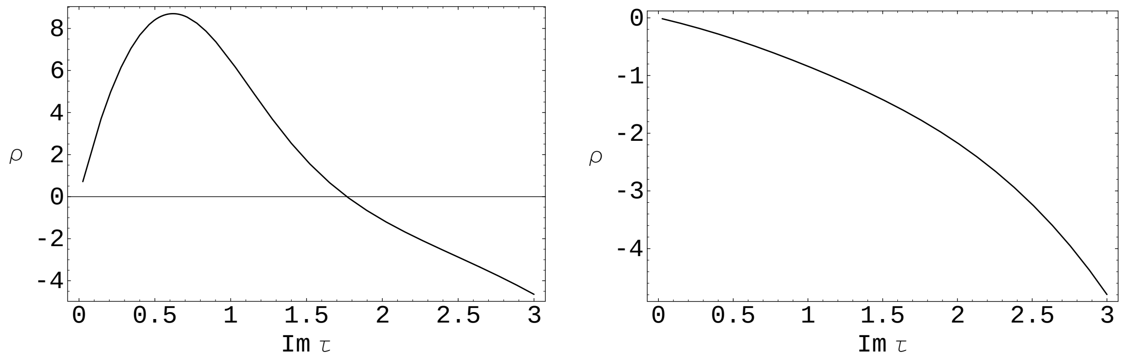

Fig. 6: Plot of the coefficient $\rho(\tau)$ in (5.8) for imaginary values of τ. It always vanishes at the origin. It also vanishes at a finite value of $\operatorname{Im}\tau$ for small m, in the left plot. When m increases the local maximum disappears and ρ becomes monotonic and negative, on the right plot for $m = 1$.

Let us look at the boundary conditions at the NS and NS' brane, coming from our exact solutions. At first order in mg_s we have

$$w = \pm m g_s (v + \bar{v}) \ . \tag{5.9}$$

Hence the embedding w grows linearly in v at infinity, as expected for a mass term in the gauge theory, while the s embedding bends logarithmically in v, as seen in (5.7), as appropriate for a running coupling. Note that, since $\gamma(\tau)$ and $\rho(\tau)$ have a smooth $\tau \to 0$ limit, we can rotate in a non-holomorphic way the dyon and monopole points as well, where the $\mathcal{N} = 2$ torus degenerates to a sphere.

Let us comment on the behavior of our solution as we increase the mass parameter m, that controls the rotation of the NS' brane away from the $\mathcal{N} = 2$ point. The coefficient $\gamma(\tau)$ is a monotonically decreasing function of τ and it drops to zero for large values of τ. There is a very interesting behavior of $\rho(\tau, m)$ as we vary m. When m is very small, ρ starts at zero, where the torus degenerates to a sphere, and has a local positive maximum, then it vanishes again at a finite value of $\tau = \hat{\tau}$. The points for which ρ vanishes have actually the same bending at infinity in the w and v directions

$$w = \pm \frac{m g_s (v + \bar{v})}{1 - m^2 g_s^2} \ . \tag{5.10}$$

However, their logarithmic bending in the s direction depends on the coefficient $\gamma(\tau)$, which is monotonic. In the case in which x_6 is non-compact, the sphere at $\tau = 0$ and the torus at $\tau = \hat{\tau}$ have different subleading logarithmic bending. Hence they have the same boundary conditions in the directions w and v but not in the direction s.

5.2. Scalar Potential

We have found that any point τ in the $\mathcal{N}=2$ moduli space can be lifted to a non-holomorphic curve. Every τ gives a different choice of boundary conditions, so we have a one parameter family of non-supersymmetric solutions to the supergravity equations. We can evaluate the M5-brane action on our solution, that is the volume of the curve, which corresponds to computing the scalar potential depending on the parameter τ. The action is infinite and needs to be regularized. The divergent part comes from the fact that the curve is non-compact, namely the embedding functions (w, v, s) have poles. A way to regularize the action is to isolate in (4.2) the term that, upon integration, is proportional to the spacetime Kahler form [6]. What is left is the integral of $G_{i\bar{j}}\partial_{\bar{z}}X^i\partial_z X^{\bar{j}}$, which vanishes for holomorphic curves. This regularization is appropriate when the boundary conditions are holomorphic. In fact, when the full curve is holomorphic, the action is zero, as expected for the energy of a supersymmetric vacuum. When the curve is non-holomorphic but the boundary conditions are still holomorphic, the action is finite and positive, and corresponds to the fact that a dynamical supersymmetry breaking vacuum has positive energy. In our case, however, the boundary conditions are non-holomorphic and this regularization, although possible, will give an infinite result anyway. Hence, we regularize the action by introducing a cutoff in the spacetime variable v_0. At this scale we specified the boundary conditions for the B period of ds, the running gauge theory coupling at the cutoff scale. The action of the fivebrane in the eleven dimensional supergravity approximation is (4.1), that we can rewrite in our simple case of flat metric as

$$A = \frac{1}{g_s^2}\int_\Sigma \left(dv \wedge *d\bar{v} + dw \wedge *d\bar{w} + g_s^2 ds \wedge *d\bar{s}\right) \ . \tag{5.11}$$

The contribution to the potential coming from w and v coordinates, which is usually subleading, in this case must be taken into account, since all of the terms are of the same order. It is straightforward to evaluate the integral (5.11) by using the Riemann bilinear relations, properly regularized to take into account the divergences. The contribution from the A and B cycles vanish and we are left with just the integral around the marked points. The embedding functions w and v give a quadratic divergence in the cutoff v_0, whose coefficients depend on the modulus τ

$$\begin{aligned}\int_\Sigma dv \wedge *d\bar{v} =& v_0^2\left(1+\left(1-\frac{m^2 g_s^2\rho}{A}\right)^2\right) + \bar{v}_0^2\left(m^4 g_s^4 + m^4 g_s^4\left(1-\frac{\rho}{A}\right)^2\right) \ , \\ \int_\Sigma dw \wedge *d\bar{w} =& 2m^2 g_s^2(v_0^2+\bar{v}_0^2) \ ,\end{aligned} \tag{5.12}$$

where the term that do not depend on m are the $\mathcal{N}=2$ contributions and $\rho(\tau)$ and $A(\tau)$ are in (5.8) and in the Appendix B. The contribution to the action (5.11) by the s embedding function is given by

$$\int_\Sigma ds\wedge *d\bar{s}=(2+m^2\gamma)^2\ln\left(\frac{v_0}{\Lambda}\right)^2+m^4\gamma^2\ln\left(\frac{\bar{v}_0}{\bar{\Lambda}}\right)^2\ , \tag{5.13}$$

The first correction comes in at order $\tilde{m}^2=m^2g_s^2$

$$V\approx V_{\mathcal{N}=2}+2m^2g_s^2\left(v_0^2+\bar{v}_0^2-\frac{\rho}{A}v_0^2+2\gamma\ln\left(\frac{v_0}{\Lambda}\right)^2\right)\ , \tag{5.14}$$

where ρ,γ and A are functions of the modulus τ. The $\mathcal{N}=2$ potential $V_{\mathcal{N}=2}$ is just a constant term and do not depend on the modulus τ. Since the divergent part depends on τ, the various non-holomorphic curves differ by an infinite amount of energy, so we cannot compare them. Note that the quadratically divergent part of the potential depends on τ through the combination ρ/A. In Fig.6 we have shown that, for small m, there are two different values of τ such that ρ vanishes. Hence, for these two different curves (a sphere and a torus) the leading divergence in the energy is the same, however the logarithmic bending is still different, so they do not represent a metastable pair of vacua.

6. Soft terms in the gauge theory limit

In this Section we want to interpret the non-holomorphic torus we have found in terms of the gauge theory. Usually, the boundary conditions at infinity correspond in the gauge theory to the choice of the classical $\mathcal{N}=1$ superpotential. Our boundary conditions at infinity (5.9) correspond to a non-holomorphic quantity in the gauge theory, so that supersymmetry is explicitly broken by a soft term (a non-supersymmetric relevant deformation). To see this, it is more convenient to shift the embedding coordinate w so that the asymptotics at the NS brane, located at $z\sim a_2$, is the more familiar $w\sim 0$. Then, the asymptotics at the NS' brane at $z\sim a_1$ represents the rotation of the NS' brane with respect to the NS brane. As usual, we can identify the embedding coordinate $v=x_4+ix_5$ with the eigenvalues of the adjoint operator Φ in the gauge theory, by matching their $U(1)_R$ charge. To leading order in the string coupling, the NS' brane at infinity is rotated with respect to the NS brane by the amount

$$w=2mg_s(v+\bar{v})+\mathcal{O}(g_s^2)\ , \tag{6.1}$$

The gauge theory limit is given as usual by taking $g_s, l_s, \Delta L \to 0$ while keeping the Yang-Mills coupling $g^2_{YM} = g_s l_s/\Delta L$ fixed, where ΔL is the distance between the NS and NS' fivebranes. The mass $\tilde{m}$ in the gauge theory is related to the string quantities by

$$\tilde{m} = \frac{g_s m}{l_s} \ . \tag{6.2}$$

The boundary conditions (6.1) resemble a gauge theory mass term for a real component of a chiral superfield. In our $\mathcal{N} = 2$ gauge theory we only have Φ, which transforms in the adjoint representation of the gauge group $U(2)$. Since we do not want to break explicitly gauge invariance, the only field whose real part can get a mass term is the $U(1)$ part of the adjoint, that we denoted $u_1 = \mathrm{Tr}\Phi$. The deformation (6.1) corresponds to a soft supersymmetry breaking mass term $\tilde{m}$ for the scalar component of $\mathrm{Re}(u_1)$

$$\mathcal{L}_{soft} = \frac{1}{4}\tilde{m}^2 (u_1 + u_1^\dagger)^2 \ . \tag{6.3}$$

It is easy to see that such a soft mass term can be obtained by the following term in the lagrangian

$$\mathcal{L}_{soft} = \int d^4\theta Z(X, X^\dagger) u_1^\dagger u_1 + \int d^2\theta M\, u_1^2 + \text{h.c.} \ , \tag{6.4}$$

by promoting the wavefunction renormalization $Z(X, X^\dagger)$ and the bare mass M to spurions with non-zero F and D components, in the case of the real superfield Z, or only F components, in the case of the chiral superfield M [27]. This is what happens when integrating out a massive messenger sector, that couples the visible sector, represented by our $\mathcal{N} = 2$ theory, to a hidden sector X, that breaks supersymmetry spontaneously by acquiring an F-term $\langle X \rangle = \Lambda_{susy} + \theta^2 F_X$. In the simplest case we can take $Z = X^\dagger X/\Lambda^2_{susy}$. By appropriately choosing the expectation values of M and X, one can then easily reproduce the soft mass term (6.3).

Our non-supersymmetric brane configuration therefore realizes $\mathcal{N} = 2$ gauge theory in which we first softly break to $\mathcal{N} = 1$ by a superpotential term and, in a second step, we break to $\mathcal{N} = 0$ by coupling it to a hidden sector through a massive messenger interaction.

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Appendix A. Elliptic functions

The main object for the construction of the elliptic (i.e. doubly periodic) functions

$$F(z) = \ln \theta \left(\pi(z - \tilde{\tau})\right) \ , \tag{A.1}$$

where $\theta(z) \equiv \theta_3(z, q)$ is the standard Jacobi theta function that has a zero at $-\pi\tilde{\tau}$ where $\tilde{\tau} = \frac{1}{2}(\tau + 1)$.[7] Hence, $F(z) \sim \ln z$ at $z \sim 0$. The n-th derivative of $F(z)$ has an n-th order pole at $z = 0$, so let us introduce the notations

$$F_i^{(n)} = \partial_z^n F(z - a_i) \ , \tag{A.2}$$

and the $F_i^{(n)}$ are elliptic for $n > 1$ and have the following monodromies for $n = 0, 1$

$$\begin{aligned} F_i(z+1) &= F_i(z), \\ F_i(z+\tau) &= F_i(z) + i\pi - 2\pi i(z - a_i), \\ F_i^{(1)}(z+1) &= F_i^{(1)}(z), \\ F_i^{(1)}(z+\tau) &= F_i^{(1)}(z) - 2\pi i. \end{aligned} \tag{A.3}$$

The $F_i^{(n)}$ also have nice properties under $z \to -z$

$$\begin{aligned} F(-z) &= F(z) - 2\pi i z + i\pi, \\ F^{(1)}(-z) &= -F(z) + 2\pi i, \\ F^{(n)}(-z) &= (-1)^n F(z) \qquad n > 1 \ , \end{aligned} \tag{A.4}$$

and, most importantly, their half period value is zero for the odd derivatives

$$F^{(2n+1)}(\tau/2) = 0 \ . \tag{A.5}$$

The asymptotic expansion of $F^{(1)}(z)$ around the origin is

$$F^{(1)}(z) = \frac{1}{z} + i\pi - 2\eta_1 z - \frac{g_2 z^3}{60} + \mathcal{O}(z^5) \ . \tag{A.6}$$

The basic Weierstrass elliptic function is defined as

$$\wp(z) = \frac{1}{z^2} + \sum_{m,n=-\infty}^{\infty} \left(\frac{1}{(z - (m + \tau n))^2} - \frac{1}{(m + n\tau)^2} \right), \qquad (m,n) \neq (0,0) \tag{A.7}$$

[7] We follow here the convenient notations and conventions in [19][28], to which we refer the interested reader.

and the Weierstrass zeta and sigma functions are defined by $\wp(z) = -\partial_z \zeta(z)$ and $\zeta(z) = \partial_z \ln \sigma(z)$. The Weierstrass functions are related to $F(z)$ as follows

$$\begin{aligned} F(z) =& \ln[\sigma(z)\theta'(\tilde{\tau})] - \eta_1 z^2 + i\pi z \ , \\ F^{(1)}(z) =& \zeta(z) - 2\eta_1 z + i\pi \ , \\ F^{(2)}(z) =& -\wp(z) - 2\eta_1 \ . \end{aligned} \tag{A.8}$$

The Weierstrass function satisfies the differential equation

$$\partial_z \wp(z)^2 = 4\wp(z)^3 - g_2 \wp(z) - g_3 \ . \tag{A.9}$$

It proves useful to rewrite some of these objects in terms of Jacobi theta functions

$$\begin{aligned} \wp(\tau/2) =& -\frac{\pi^2}{3}\left[\theta_2^4(0) + \theta_3^4(0)\right] \ , \\ \eta_1(\tau) =& \zeta(\frac{1}{2}) = -\frac{\pi^2}{6}\frac{\theta_1'''(0)}{\theta_1'(0)} \ , \\ g_2(\tau) =& \frac{2\pi^4}{3}\left[\theta_2^8(0) + \theta_3^8(0) + \theta_4^8(0)\right] \ , \end{aligned} \tag{A.10}$$

Appendix B. The parametric $\mathcal{N} = 2$ curve

The brane configuration

$$\begin{array}{ccccccccc} & x_0 & x_1 & x_2 & x_3 & v & x_6 & x_7 & w \\ NS & \bullet & \bullet & \bullet & \bullet & \bullet & \times & \times & \times \\ NS' & \bullet & \bullet & \bullet & \bullet & \bullet & \times & \times & \times \\ D4 & \bullet & \bullet & \bullet & \bullet & \times & \bullet & \times & \times \end{array} \tag{B.1}$$

describes $\mathcal{N} = 2$ gauge theory with $U(N)$ gauge group. Its lift to M theory[8] [30] is an M5-brane wrapping the holomorphic curve of genus $N-1$

$$\Sigma_c : \qquad \left\{ \begin{array}{c} t^2 - 2tP_N(v, u_r) + 4\Lambda^{2N} = 0 \ , \\ w = 0 \ , \end{array} \right. \tag{B.2}$$

plus a bunch of disconnected complex lines, in the case we have also flat spectator NS' branes. Let us consider the case in which the gauge group is $U(2)$, i.e. we have two D4-branes. In this case the curve is a torus. We would like to give a parametric description

[8] For related work, see [29].

of (B.2) in the z coordinate, [9] by using elliptic functions. The embedding functions s and v are holomorphic and completely fixed by their periods and their asymptotic boundary conditions to

$$\begin{aligned} s_{SW}(z) =& 2\left(F(z-a_1) - F(z-a_2)\right) - 2\pi i z + s_0 \ , \\ v_{SW}(z) =& A\left(F^{(1)}(z-a_1) - F^{(1)}(z-a_2) - i\pi\right) + \frac{1}{2}u_1 \ , \end{aligned} \tag{B.3}$$

and $w(z) = 0$, while the B-period constraint (4.15) fixes $a = -\tau/2$. We would like to find the map between the parametric quantities τ, A, s_0 in (B.3) and the physical quantities u_1, u_2, Λ in (B.2). Noting that $t = e^{-s}$, by plugging (B.3) into (B.2) with characteristic polynomial $P_2(v) = v^2 - u_1 v - u_2 + \frac{1}{2}u_1^2$ we eventually find the exact map between the parametric and the physical quantities

$$\begin{aligned} A^2(\tau) =& -\Lambda^2\left(12\wp(\tau/2)^2 - g_2(\tau)\right)^{-\frac{1}{2}} \ , \\ u_2(\tau) =& -3\wp(\tau/2)A^2(\tau) + \frac{u_1^2}{4} \ . \end{aligned} \tag{B.4}$$

The parametrization of the moduli space using τ is actually a multiple covering. A part from the obvious symmetry $u(\tau+2) = u(\tau)$, there are also the reflection symmetries [31] $u(\tau+1) = -u(\tau)$ and $u(-\bar{\tau}) = \overline{u(\tau)}$. The three punctures are at $u(\tau=0) = \Lambda^2$, $u(\tau=1) = -\Lambda^2$ and $u(\tau=\infty) = \infty$.

Appendix C. Virasoro condition for the non-holomorphic torus

In this Appendix we give some details about the computation of the exact non-holomorphic solution in (5.2), (5.3) and (5.4).

Let us discuss the conditions on the harmonic embedding coordinate s. The most general elliptic and harmonic function satisfying the period conditions (4.11), (4.12) is

$$\begin{aligned} s =& (2+m^2\gamma)(F_1 - F_2) - i\pi(2-m^2\delta)z \\ & + m^2\gamma(\overline{F_1} - \overline{F_2}) - i\pi m^2\delta\bar{z} \ , \end{aligned} \tag{C.1}$$

where γ, δ, A are constant coefficients to be fixed. The condition (4.12) gives

$$-4a - 2\tau + m^2\delta(\tau - \bar{\tau}) - 2m^2\gamma(a - \bar{a}) = 0 \ , \tag{C.2}$$

[9] We use similar techniques to [28][19], who studied the $\mathcal{N}=1$ case.

which fixes the location of the two marked points on the torus as in (4.15) and is satisfied by $a = -\tau/2$ as in the $\mathcal{N} = 2$ case, with $\delta = -\gamma$. The B-period in (4.12) then fixes the dependence of τ on the running coupling of the gauge theory at the cutoff scale. The parameters ρ, γ, δ in our ansatz (5.2), (5.3) and (C.1) are going to be fixed by solving the Virasoro condition

$$g_s^2 \partial s \partial \bar{s} + \partial v \partial \bar{v} + \partial w \partial \bar{w} = 0 \ . \tag{C.3}$$

As explained in the main text, we just need to expand (C.3) around a pole, say $z = a_1$, and impose that the coefficients of the poles of different degrees and the constant term in the expression separately all vanish. The quartic pole cancels automatically, while we have again three complex equations coming from the double pole, the simple pole and the constant term. At the special point $a = \tau/2$, solution of (C.2), we have $\delta = -\gamma$ and the odd derivatives $F^{(2n+1)}(\tau/2) = 0$. The equations simplify and allow to solve for the real and imaginary parts of the coefficients in the game, namely γ, ρ. We want the solutions to satisfy the requirement that, in the limit $m \to 0$, we recover the $\mathcal{N} = 2$ solution (4.8), so γ, ρ must be finite. It turns out that there is a unique solution satisfying these requirements.

The exact solution is

$$\begin{aligned} \gamma &= -\frac{1}{m^2} + \frac{1}{m^2}\left(\frac{\alpha(\tau,m,g_s) + (1-m^2g_s^2)A^2\beta(\tau,m,g_s)^{\frac{1}{2}}}{8g_s^2(\wp + 2\eta_1)}\right)^{\frac{1}{2}} , \\ \rho &= \frac{1}{(m^2g_s^2-1)}\left(4A + \frac{\gamma(2+m\gamma)}{Am(\wp+2\eta_1)}\right) , \end{aligned} \tag{C.4}$$

where we introduced the elliptic functions

$$\begin{aligned} \alpha(\tau,m,g_s) =& 8g_s^2(\wp+2\eta_1)\left(1-4m^2A^2(\wp+2\eta_1)\right) - (1-m^2g_s^2)^2A^2h(\tau) , \\ \beta(\tau,m,g_s) =& (1+m^2g_s^2)^2h(\tau)^2 - 4m^2g_s^2(h(\tau)-8(\wp+2\eta_1)^2)^2 , \\ h(\tau) =& 24\wp(\tau/2)^2 + 24\wp(\tau/2)\eta_1(\tau) - g_2(\tau) . \end{aligned} \tag{C.5}$$

The solution can be expanded to first order in the mass

$$\begin{aligned} \gamma &\sim -8A^2\frac{(\wp+2\eta_1)^3}{h(\tau)} + \mathcal{O}(m^3), \\ \rho &\sim -4A\left(1-4\frac{(\wp+2\eta_1)^2}{h(\tau)}\right) + \mathcal{O}(m^3) , \end{aligned} \tag{C.6}$$

where $\wp$ is the Weierstrass $\wp$-function evaluated at $\tau/2$ and η_1 and g_2 are and some standard coefficients

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